ProMath 2017 conference and the annual meeting of the working group Hungary

Second Announcement

Conference place: Eötvös Loránd University, Faculty of Science, Budapest, Pázmány Péter sétány 1.c, Room Turán Pál (Room 3.219)

PROGRAM

30th August 2017

12.00-13.00 Registration

13.00-13.15 Opening

Each presentation contains 20 minutes talk + 10 minutes discussion

13.15 – 15.45 **Chair: Ödön Vancsó**

Friedlander, Alex: Criteria for "Good Problems"

Scharnberg, Sarina: Qualities of Successful Problem-solving Teachers

Aktas, Fatma Nur & Yakici-Topbas, Esra Selcen: Cognitive-Metacognitive Process through Mathematical Problem Solving in a Small Group: Dynamic Geometry Systems or Paper-Pencil Environments?

Ambrus, Gabriella & Kónya, Eszter: Solving of Real Situations Based Problem - Experience with Teacher Training Students

Katona, Dániel: Web of Problem Threads in the Pósa Method in Hungary

15.45 – 16.15 Coffee break

16.15 – 18.15 **Chair: Erkki Pehkonen**

Rott, Benjamin: Problem Solving in the Classroom: How do teachers organize lessons with the subject problem solving?

Yakici-Topbas, Esra Selcen & Aktas, Fatma Nur: Prospective Secondary Mathematics Teachers' Prompting in the Problem-Solving Process: The Context of Metacognitive Strategies

Kovács, Zoltán: Math Teacher trainees facing with the "What-If-Not" strategy - a case study

Szűcs, Kinga: Problem Solving Teaching in Inclusive Classrooms

19.30 Dinner on the ship A38 (Danube)

31st August 2017

8.30 – 10.30 **Chair: Gabriella Ambrus**

Pehkonen, Erkki: Developing of Teaching via Problem Posing and Solving

Papadopoulos, Ioannis & Sekeroglou, Ioanna: Types of control in collaborative problem solving

Ohlendorf, Meike: Pólya's stage Looking back in the Classroom

Wintsche, Gergely: The Usefulness of Independent Teacher Feedbacks in the new Mathematics Textbooks

10.30 – 11.00 Coffee break

11.00 – 13.30 **Chair: Günter Graumann**

Kuzle, Ana: Analysis of the Development of one Teacher's knowledge for Teaching Problem Solving

Gosztonyi, Katalin: The Role of Classroom Dialogues in the Hungarian IBME Tradition

Kabael, Tangül & Yayan, Betül: Preservice Middle School Mathematics Teachers' Questioning Skills in problem solving process and Their Conceptions of Problem and Problem Solving

Berta, Tünde: The Competences of Students Majoring in Teacher Training in Mathematics Problem Solving, Lessons Learned from Mathematics Monitor in Slovakia

Yayan, Betül: Performances of Eighth Grade Students of Singapore, the United States and Turkey in Ratio and Proportion Problems

13.30 – 15.00 Lunch

15.00 Walking in Budapest: "Hidden treasures of Budapest" (about 3 hours)

1st September 2017

8.15 – 10.15 **Chair: Benjamin Rott**

Assmus, Daniela, Förster, Frank & Fritzlar, Torsten: Similarities between Mathematical Problems from the Perspective of Primary Students

Vargyas, Emese: Geometric Transformations as a Tool in Problem Solving

Leinonen, Jorma: Roles of Understanding in Problem Solving and Learning in Mathematics

Graumann, Günter: Problems in the context of the Thales circle

10.15 – 10.45 Coffee break

10.45 – 13.05 **Chair: Ana Kuzle**

Szanyi, Gyöngyi: The Effect of an Improvement Program on the Formation of the Function Concept

Gunčaga, Ján: Some Aspects of Problem Solving in Historical Mathematical Textbooks

Vancsó Ödön: Problem Sequences in Uncertain Situations

Vásárhelyi Éva: Imaginary Report with András Ambrus

13.05-13.35 Closing, ProMath 2018 Conference

13.35 Lunch

15.00-16.00 Sitzung des Arbeitskreises Ungarn (Informationen, Aktualitäten) – Interessierte Kollege sind herzlich eingeladen

Excursion (Free option)

Walking in Budapest: “Hidden treasures of Budapest” Guided tour, about 3 hours. Participation fee 15 Euros/person.

Please a feedback until 15 July if you are interested in it.

Conference dinner

Ship A38 Restaurant (Danube River) Close to the conference place. Everybody pays her/his order.

Conference abstracts

On the meeting there will be no hard copy version of abstracts, only this electronics one!

Aktas, Fatma Nur & Yakici-Topbas, Esra Selcen, Gazi University, Turkey
Cognitive-Metacognitive Process through Mathematical Problem Solving in a Small Group: Dynamic Geometry Systems or Paper-Pencil Environments?
fnuraktas@gmail.com / selcenyakici@gmail.com

Problem solving is a thinking process that a solver tries to make sense of a problem situation using mathematical knowledge which the solver has and attempts to obtain new information some endeavors (Lester & Kehle, 2003). Cognitive and metacognitive process have important role for successful mathematical problem solving and maximum student involvement (Artz & Armour-Thomass, 1992). Another factor that affect problem-solving positivel is using small groups. Small-group approaches increase the solver's problem solving achievement and improve abilities in the mathematical problem-solving process (Davidson & Kroll, 1991). Small problem-solving groups support natural settings for interpersonal monitoring and regulating of members and these settings are liable for positive effects observed in the groups (Artz & Armour-Thomass, 1992). Considering the importance of these factors for problem solving in mathematics education, we focused on cognitive and metacognitive process of a small group in two different environments in this study.
The purpose of the study is to delinate and to identify dynamic geometry software (GeoGebra) and paper-pencil environments' effects of cognitive and metacognitive processes within small-group heuristic problem solving in mathematics. Changing the environment may provide students to question for additional solutions and to benefit a deeper understanding of the problem (Koyuncu, Akyuz & Cakiroglu, 2015). Also, we aimed to investigate individuals' engagement in mathematical problem solving in small-group settings the cognitive processing context. Through this investigation, we expected to learn more about how levels of cognitive processes interact and contribute to the successful outcomes of problem solving within small groups.

The study was designed as a case study. Participants were selected based on their problem solving performance in a mathematics education course. Then, a heterogeneous small group was formed with four prospective mathematics teachers. Two different problems were given to this small group. While participants solved the first problem with using GeoGeba and then paper-pencil environments, they solved the second problem firstly using paper pencil and then geogebra environment. The problems were related to reflections in transformations. The reason for selecting these problems was provide the variety of problem solving approaches (calculus geometry etc.).They also included real life situations, and could be solved with using dynamic geometry software.

The data sources for this study were participants' written notes, solutions in geogebra environment and video recordings of small group. The cognitive and metacognitive processes in data will be analyzed by Arzt & Armour-Thomas's (1992) framework for protocol analysis of mathematical problem solving in small group. Findings will be presented with respect to problem-solving episodes as reading, understanding, exploring, analyzing, planning, implementing, watching and verifying. As an expected result might be the positive effect of

cognitive processes on prospective teachers' content knowledge. This study also might provide prospective teachers with experiencing of using both paper-pencil and dynamic geometry software to raise awareness about how a student behave in problem solving process.

Ambrus, Gabriella Eötvös L. University Budapest, Hungary & Kónya, Eszter University Debrecen, Hungary
Solving of Real Situations Based Problem - Experience with Teacher Training Students
ambrusg@cs.elte.hu, eszter.konya@science.unideb.hu

An important task is for the future teachers to teach their students for the use of their mathematical knowledge even in everyday (real) situations. This task requires often from the students a change of view- according to our tests. Last year we started to work out a program which can (may) help them to a development in this subject. In the presentation we show some details of this program and we are discussing our first experience.

Assmus, Daniela (*), Förster, Frank (**) & Fritzlar, Torsten (*)
(*) University of Halle-Wittenberg, Germany / (**) Technical University of Brunswick, Germany
Similarities between Mathematical Problems from the Perspective of Primary Students
daniela.assmus@paedagogik.uni-halle.de, f.foerster@tu-bs.de, torsten.fritzlar@paedagogik.uni-halle.de

The construction and use of analogies is regarded as an important problem-solving strategy not only in the field of mathematics. From numerous studies, however, it is also known that this strategy is especially challenging for primary school children. One difficulty that must be overcome is to distinguish between superficial and mathematically relevant (structure-related) similarities of problems.

Therefore, in qualitatively oriented case studies we investigated what primary students pay attention to when assessing similarities between mathematical problems. Corresponding knowledge is important for conceiving further researches on the construction and use of analogies as well as teaching activities for analogy. First results will be presented in the lecture.

Berta, Tünde, Selye J. University Komárno, Slovakia
The Competences of Students Majoring in Teacher Training in Mathematics Problem Solving, Lessons Learned from Mathematics Monitor in Slovakia
btunde@yahoo.com, bertat@ujs.sk

The focus of my work is on the presentation of results of tests conducted among students of Hans Selye University majoring teacher training. The test included the tasks of official mathematics Monitor of Slovak Republic for 5th grade students, the aim of which is to test the knowledge of mathematics acquired in the primary (1st to 4th grade).

The test has been conducted among students from different years, the major aim of the test has been the evaluation of the competences of mathematics at primary level of future teachers and kindergarten teachers. Different tasks and their solution is going to be presented in my work, special attention is paid to the solution of problematic tasks – primarily from combinatorics and compound word problems and to plausible shortcomings of cogitation of students in the curriculum of teacher training at Selye University in Slovakia and in elementary school curriculum in Slovakia.

Friedlander, Alex, Weizmann Institute of Science, Tel Aviv, Israel
Criteria for "Good Problems"

I would like to consider problem solving as a basic instructional activity that takes up a considerable part of learning mathematics for ***all*** students – rather than its being a "top activity" aimed at "top students".
I plan to present two mathematical problems, and analyze them according to several sets of criteria (e.g., Schoenfeld, Friedlander & Arcavi, Even) in order to understand some of the characteristics that make a "good problem".

Gosztonyi, Katalin, Eötvös L. University, Budapest
The Role of Classroom Dialogues in the Hungarian IBME Tradition
katalin.gosztonyi@gmail.com

Hungarian mathematics education is well known for having a strong tradition in problem solving and inquiry based teaching. One of the important characteristics of this tradition is the central role of dialogues between students or between teacher and students: mathematical discovery process being considered as a collective, social activity (Gosztonyi 2015, 2016). In this presentation, I will show on some concrete examples how the teacher can intervene in order to guide the process of collective discovery, leaving however an important responsibility to students in the construction of mathematical knowledge.

Graumann, Günter, University of Bielefeld, Germany
Problems in the Context of the Thales Circle
og-graumann@web.de

We start with a segment AB and a circle which has AB as diagonal. If we then choose a point C of this circle with C ≠ A, B and move C along the circle we always get a right-angled triangle. This is the statement of the well-known theorem of Thales.
In connection with this theorem we get some interesting problems if look out for other measures and special points of these right-angled triangles. Which traces are built by the midpoint of the inner circles or gravity center? How does the area of these triangles or the length of its height change? How does the square determined by A, B, C change? These examples concern grade 8 to 12.

Such activities encourage the students to see a well-known theorem in a broader context and to look out for possible problem posing in a given problem field. Besides all these questions can be very good supported by a Dynamic Geometry Software and the students can learn that some classical problems also can be seen in a dynamic way.

Gunčaga, Ján, Catholic University in Ružomberok, Slovakia
Some Aspects of Problem Solving in Historical Mathematical Textbooks
jan.guncaga@ku.sk

This contribution makes an excursion into the history of mathematics, which is used in school mathematics. We would like to describe some components of mathematical notions and problem solving developed by Jakub Kresa (1648-1715). We present also some approaches in the problem solving by Franz Močnik (1814-1892). This historical approaches have connection to nowadays mathematics, because many international studies such PISA support problem solving and using real-life oriented problems in mathematics education.

Kabael, Tangül & Yayan, Betül, Anadolu University, Turkey
Preservice Middle School Mathematics Teachers' Questioning Skills in Problem Solving Process and Their Conceptions of Problem and Problem Solving
tuygur@anadolu.edu.tr

The purpose of this qualitative study is to investigate the questioning skills and conceptions of four volunteer preservice middle school mathematics teachers within the context of problem solving. The data of the study was collected through clinical interviews that the participants conducted with eighth grade students and researchers conducted with participants. The findings revealed that three of the four preservice teachers could not use appropriate questioning in the problem solving process and none of them could make the formal definition of problem and problem solving phases. Since problem solving has a crucial place in the mathematics education, the questioning skills of pre-service mathematics teachers has a particular importance in mathematics teaching. On the other hand, it was seen that only one out of four preservice middle school mathematics teachers could use the appropriate questioning that would enable her student to reach the intended generalization in the problem by supporting her mental process in the problem solving process. Moreover, only this participant proved to have the terminological knowledge in the mentioned context with her conceptions. The participants, who could not use appropriate questioning, was seen that they did not have what a problem mean and how the problem solving process is. Therefore, the results indicated that e conceptions of the preservice teachers within the context of problem solving and their questioning skills in problem solving process were consistent.

Katona, Dániel, Budapest, Hungary
Web of Problem Threads in the Pósa Method in Hungary
danikatona@gmail.com

The Pósa method is one of the most prominent examples of current Hungarian practice of discovery learning of mathematics for gifted upper-primary and secondary school students, who are involved in mathematical problem-solving processes at week-end math camps. However, in the framework of the Content Pedagogy Research Program of the Hungarian Academy of Sciences new experimental programs have been launched too, in order for broadening the set of target students, to be applied with less talented students as well, and spreading the method around the country.
The Pósa method is highly characterized by the use of *web of problem threads*, which are to trigger the problem-solving process. Besides the enumeration of the supposed – as currently researched – didactical principles of the method, a sample from a problem thread is to be presented, with mostly geometric and combinatorial problems relating to constructions of unit cubes. By the analysis of the problems and their solutions, the connections between the problems are to be revealed, and the so called *kernels of the threads*, as principal means of triggering the discovery learning process, are to be studied.
The main purpose of the presentation is to introduce the aforementioned mathematics didactic concepts – problem threads, web of problem threads and their kernels – as basic notions of a didactical theory for the analysis of teaching mathematical problem-solving, that is presently under formulation by the author and his colleagues, Péter Juhász and Gábor Szűcs, in the MTA-Rényi Research Group on Discovery Learning in Mathematics at the Hungarian Academy of Sciences.

Szűcs, Kinga, University of Jena, Germany
Problem Solving Teaching in Inclusive Classrooms
kinga.szuecs@uni-jena.de

In the last few years, the concept of inclusion has become more and more prevalent in mathematics classrooms in Germany and elsewhere. This is not just an opportunity, but also a big challenge for teachers of mathematics. The Friedrich Schiller University Jena is a participant in the national programme to improve the quality of teacher training *("Qualitätsoffensive Lehrerbildung"),* which is funded by the German Federal Ministry of Education and Research. Within the framework of the subproject "Media in mathematics education", the use of new technologies in the mathematics classroom – especially, but not only digital media – is examined. The main focus in the academic year 2016/17 is to discover how new technologies can help to deal with the heterogeneity of the students, especially with a combination of visually handicapped and sighted students.

In the talk, some first results from the academic year 2016/17 will be shown as examples. For some concrete mathematical topics (linear system of equations, geometrical probability) concrete problem-solving teaching units were worked out according to Pólya, and one of them was tested in a mathematics classroom in Jena. The talk will present the teaching units, and report on practical experiences.

Kovács, Zoltán, University of Nyíregyháza, Hungary
Math Teacher Trainees facing with the "What-If-Not" Strategy - a Case Study
kovacs.zoltan@nye.hu

This study is guided by the question: Can a systematic instruction in problem posing strategies increase teachers' flexibility in the way they look at textbook problems? The participants in the study consisted of group of math teacher trainees, in the last semester of their training. They got experiences with problem posing strategies during their training. After solving a textbook problem students explored the problem more deeply by putting questions using the "What-If-Not" strategy. Three different solutions appeared in the classroom. I analyse these approaches related to Polya's steps of problem solving.

Kuzle, Ana, University of Potsdam, Germany
Analysis of the Development of one Teacher's knowledge for Teaching Problem Solving
kuzle@uni-potsdam.de

In this talk I demonstrate the possibilities for developing materials using design-based research (DBR) focusing on the development of mathematics teachers' knowledge for teaching problem solving. In accordance with DBR, the design process was informed by theoretical basis with respect to models of teachers' knowledge for teaching problem solving (e.g., Chapman, 2015, 2016). More concretely, I focus on two question: How does material informed by models of teachers' knowledge for teaching problem solving foster or hinder the implementation of problem solving in mathematics? How does teacher's knowledge supported by problem solving materials develop through different implementation cycles?

The results are based on four DBR-cycles with one teacher through which I demonstrate how the theoretical ideas got implemented and what design elements fostered or hindered the implementation. On this basis I offer suggestions for how material related to development of teachers' knowledge for teaching problem solving might be designed to support their diverse needs.

Leinonen, Jorma, University of Lapland, Finland
Roles of Understanding in Problem Solving and Learning in Mathematics
jorma.leinonen@ulapland.fi

According to Finnish curriculums the aim of learning of subjects is to understand structures of knowledge in subjects and develop understanding. After NTCM (2000, 20) students must learn mathematics with understanding, actively building knowledge from experience and prior knowledge. Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. It gives a knowledge context and tools for thinking and learning processes. Problem solving is a tool for mathematical thinking and understanding. Problem solving includes a static and dynamic part of understanding. The former gives framework to process the information. The function of the dynamic part builds a new knowledge from experience and prior knowledge. The act of understanding refers to two different categories of cognitive process: interpretation and synthesis (cf. Leinonen 2011, ProMath, Jena).

In Polya's model of problem solving understanding is the first step. One must at first interpret the task to comprehend (identification) the main parts in the situation: the unknown, the data and the connections between them. Understanding and reasoning are complementary entities in problem solving. The act of reasoning together with conceptual understanding generates a solution to the problem and in addition makes operational and factual information and new strategies. After synthesis the learner has to integrate the new information into his/her cognitive structure. The new structure of proceptual understanding gives reference frames and tools for interpretation in new tasks.

Ohlendorf, Meike, University of Brunswick, Germany
Pólya's Stage Looking back in the Classroom
m.ohlendorf@tu-bs.de

Pólya's stage "looking back" is often said to be the most important stage in problem solving. "Looking back" can mean merely checking the solution, but reflecting on the problem-solving process can also lead to alternative solutions or make aware mistakes as well as general strategies or extensions of the problem. In this way, the stage "looking back" can be seen as a link between problem solving and problem posing.

Despite the importance of "looking back", the data of an empirical study conducted in German secondary schools suggests that these reflections are rather neglected in the classroom. The aim of this study is to characterise in which way the 14 participating secondary teachers make their students engage in looking back activities. This presentation will describe theoretical background of the stage "looking back" as well as provide an insight into initial results of the empirical study.

Papadopoulos, Ioannis & Sekeroglou, Ioanna, Greece
Types of Control in Collaborative Problem Solving
ypapadop@eled.auth.gr

The issue of '*control'* is a way to talk about metacognition in problem solving and constitutes an essential procedure that gives to the students the opportunity for mental development and growth to an efficient mathematical problem solver (Schoenfeld, 1987). In this paper we aim to examine the different types of control used by six pairs of pre-service teachers during their effort to solve two problems (an arithmetical and a geometrical one). Working in pairs is useful in problem solving since the solvers communicate their thoughts and this triggers control (Ambrus & Barczi-Veres, 2016) The analysis of the collected and transcribed protocols reveal that there are two types of control: Global and Context-based control actions. Global control actions are applied to all types of problem no matter their topic. Such actions are related to identifying errors, checking whether the conditions of the problem are satisfied, whether the result or the chosen strategy is reasonable, to check the correct understanding of the problem, to verify the final result. Context-based control actions are closely related to the specific problem and make sense only in case they are applied to its solution. We also wanted to examine what triggers control actions in a

collaborative problem solving. It seems that some of the reasons are: the case of problems with multiple solutions, conflicting results obtained separately by each member of the pair, the unreasonableness of the obtained result, and finally, the inability to proceed further the solution (in this case the solvers often attribute this to a mistaken action made earlier).

Pehkonen, Erkki, University of Helsinki, Finland
Developing of Teaching via Problem Posing and Solving
erkki.pehkonen@helsinki.fi

This paper describes the concepts 'problem solving' and 'problem posing' that are today in the focus of developing mathematics teaching. In addition to some theoretical considerations a couple of examples are given and discussed.

Rott, Benjamin, University of Cologne, Germany
Problem Solving in the Classroom: How do Teachers organize Lessons with the Subject Problem Solving?
benjamin.rott@uni-koeln.de

Despite the importance of problem solving for mathematics and mathematics teaching, many teachers are not familiar with this subject. How do teachers plan and organize lessons with the subject "problem solving"? What factors can be used to explain their behavior? In the study at hand, several teachers' lessons have been filmed and analyzed with regard to the teachers' actions. A two-dimensional coding scheme is presented that helps to categorize and compare those actions. This categorization reveals interesting correlations between the teachers' actions and their beliefs regarding mathematics and problem solving.

Scharnberg, Sarina, University of Lueneburg, Germany
Qualities of Successful Problem-solving Teachers
scharnberg@leuphana.de

In recent years, problem-solving became one of the key competences of (mathematics) teaching, particularly because of its acknowledged value for students in modern society. With the introduction of the German educational standards for mathematics in 2003, problem-solving became one of the five process related competences (cf. KMK 2003) and therefore a mandatory component of mathematics teaching in Germany today. Nonetheless, the reality paints a sobering picture: Problem solving is hardly taught in German mathematics classrooms and German students' problem-solving performance is particularly low in comparison to their performances in science, reading and general mathematics (cf. OECD 2014). While due to a number of factors, a large role for this is played by the teacher's competence in this area (cf. Fritzlar 2004). Consequently, it is important to place a particular focus on teaching problem-solving in the context of student teacher education in order to improve the qualities of future problem-solving teachers.

As a part of the Initiative for Excellence in Teacher Education, which is funded by the German Ministry for Education and Research, a so-called *Community of Practice* (comprised of mathematics teachers, professors, and researchers) of the *Zukunftszentrum Lehrerbildung* at the Leuphana University Lueneburg develops a course on how to plan and carry out a problem-solving lesson initiating a theory-to-practice-approach. Therefore, the course design does not just reflect research regarding models and teaching concepts for problem-solving (cf. e.g. Bruder; Collet 2011) on a theoretical level but it also enables student teachers to plan and reflect on their own problem-solving lessons in cooperation with teachers from the *Community of Practice*. With regard to their own teaching, these teachers, in turn, benefit from the ready to use material and lesson plans designed by the student teachers.

This presentation will provide an input on the theoretical background as well as the course structure. Furthermore, it will enable the identification of the qualities of teachers who successfully convey principles of problem-solving, as the long-term goal of this course is not just to improve the student teachers' planning competences (including material preparation), but to also improve their teaching quality.

Szanyi, Gyöngyi, University of Debrecen, Hungary
The effect of an improvement program on the formation of the function concept
szanyi.gyongyi@science.unideb.hu

The concept of *function* has a fundamental importance in learning mathematics. Like most mathematical concepts, the function concept can not be acquired in one step, that is why its formation takes a long process. Hence, laying the foundation of this basic concept should be initiated with thorough preparatory work before its introduction to pupils.
Studies were made in the light of function concept formation processes in the Ukrainian and Hungarian curriculum. According to the results Hungarian texbooks and curriculum give a more targeted preparatory work from the elementary school to establish the concept of function. The investigations that were carried out in one class in both among Hungarian and Ukrainian students in 6th and 7th grades concluded that incorporating the source of the concept into the teaching process strengthens the formation of the correct function schema. According to the experiences we prepared and implemented an improvement program on the preparation of the function concept among 6th grade students in Ukraine.
In our presentation we are examining the effect of the improvement program on the formation of the function concept after 1 year (in 7th grade).

Vancsó, Ödön, Eötvös L. University of Budapest, Hungary
Problem Sequences in Uncertain Situations
vancso.odon@gmail.com

Some interesting „Game of Chance" problems organised in a sequence will be shown in this presentation. The idea of choosing these tasks and problems will be explained why they are suitable for a wider experiment. The problems were collected first in an EU project DQME-I, and tried out in some schools. We are planning now a bigger project on this field by participation of some teachers and classes using video typing of lessons and analyzing the students' solutions.

Vargyas, Emese, University of Mainz, Germany
Geometric Transformations as a Tool in Problem Solving
vargyas@uni-mainz.de

Transformations, such as translations, rotations, reflections and combinations of these, play an important role in school geometry. At the beginning they occur mainly as the objects of different investigations: how to recognize transformations, how to work out their definitions, how to discover their properties, etc. Students do generally well on these tasks. Problems arise if transformations should be used as a tool for solving problems in which the right transformation has to be identified first. Based on an elementary geometric problem, the presentation provides an example how to teach students to find appropriate transformations in such cases.

Wintsche, Gergely, Eötvös L. University Budapest, Hungary
The Usefulness of Independent Teacher Feedbacks in the New Mathematics Textbooks
wintsche@cs.elte.hu

I have had the opportunity to develop textbooks and other learning tools within the framework of the Social Renewal Operational Program (SROP) 3.1.2/ B-13. The project involved not only the creation of new, but also the exploration of teaching practice in connection to the usage of the new textbooks, collecting and analyzing feedback. My responsibility was the math textbooks and practicing books for from grade 5^{th} up to grade 8^{th}.
More than 100 000 pieces of feedback were collected for the 5^{th}-8^{th} grade textbooks during the project. We created feedback categories e.g. errors, misprints, theoretical problems, didactical problems, suggestions, good practices, wishes, etc.. In this paper I focus on the official feedback collected from teachers. I will present a short summary about the changes in the revised textbooks influenced by the feedback.

Yakici-Topbas, Esra Selcen & Aktas, Fatma Nur, Gazi University, Turkey
Prospective Secondary Mathematics Teachers' Prompting in the Problem-Solving Process: The Context of Metacognitive Strategies
selcenyakici@gmail.com

Metacognitive prompting is defined as a stimulus which is externally produced activating reflective cognition, or kindles strategy use with the objective of improving a learning or problem-solving outcome by Hoffman & Spatariu (2007). Awareness of task characteristics, performance strategies and evaluation of outcomes are evoked by prompting (Butler & Winne, 1995). MoNE (2013) and NCTM (2000) has put emphasis on improving students' problem solving skills in the secondary mathematics curriculum. Therefore, a teacher's competence is of importance for developing students' problem solving skills and teachers should prepare prompts in students' problem solving process to enhance learning. This competence is also expected from prospective teachers.

The purpose of this study is to examine how prospective mathematics teachers prepare metacognitive prompts during the students' problem solving process. These prompts were

analyzed in metacognitive strategic context. The study was conducted with 23 prospective mathematics teachers studying at a five-year teacher education program at the Secondary Mathematics Education Department of a public university in Turkey. Participants receiving student-centered teaching methods in a mathematics education course and familiar with Polya's (1945) problem solving process were selected.

$\sqrt{x-5} = \sqrt{x} - \sqrt{5}$

The study was designed as a case study in which the participants were asked to prepare tasks for prompting on the following algebraic problem:

'Investigate whether the following equation is true or not:

We chose this algebraic problem as it includes both the solving of linear and quadratic equations and algebraic thinking. Furthermore, it captures the kinds of relational thinking that underlie algebraic reasoning and that distinguish it from arithmetic activity, which is typically computational in nature (Kieran, 2014).

Firstly, participants completed these tasks individually; then, they discussed their prompting in small groups of three or four prospective teachers. The data for this study includes participants' written notes and video recordings of small groups. Content analysis was used for data analysis (Merriam, 1998). We used Veenman (2011) "WWW&H" rule (What to-do, Why they are beneficial, When to use them, and How to use strategies) to categorize mathematics prospective teachers' prompting. In addition, we matched Polya (1945)'s problem solving phases with these categories. Results indicated that the prompts in the category of

- 'What to do?' are generally connected to the understanding phase.
- 'Why they are beneficial?' consist of devising a plan, carrying out the plan and looking back phases. The prompts in this category are also similar to the prompts of Hoffmann and Spatariu (2007) which are 'Can your answer be checked for accuracy? What strategy are you using to solve the problems? Are these problems similar to addition in any way? What is the best method to solve the problem?'.
- 'When to use them?', all of the participants designed prompts about devising a plan phase. Prompts which were designed for this category are generally in the form of 'Have you solved similar problems before?'.
- 'How to use strategies?' matched the phase 'Carrying out the plan'.

What is more some prompts such as 'can you put these points that you found by trial and error method on the number line and generalize the other points?' can be included in both the category of 'when to use them' and 'how to use strategies'. According to the analysis, prospective teachers make their students ready to the problem by scaffolding, and then, pose the prompts that they prepared to the students. Through scaffolding such as metacognitive approaches, teachers are able to create suitable learning activities that not only engage their students in problem solving, but also facilitate their improvement (Ghasempour et al).

Yayan, Betül, Eskişehir, Anadolu University, Turkey
Performances of Eighth Grade Students of Singapore, the United States and Turkey in Ratio and Proportion Problems
byayan@anadolu.edu.tr

The concepts of ratio and proportion have an important place in middle school mathematics and more importantly understanding these concepts are key to the success in both higher mathematics and other fields of knowledge. Ratio and proportion problems involve situations in which the relationships between the quantities are multiplicative (as opposed to additive) in nature and in these problems students are require to construct two equal ratios between these quantities. The ability to solve such problems indicates the existence of proportional reasoning. In this study, performances of eighth graders from three selected countries in solving ratio and proportion problems in Trends in International Mathematics and Science Study (TIMSS) 2007 and 2015 are investigated. Three selected countries are Singapore, the United States, and Turkey. Singapore is one of the top performing countries in all TIMSS studies in mathematics. On the other hand, eighth graders from the United States do not perform as well as top performing countries however, they display average mathematics performance being above the international average. Finally, eighth graders from Turkey perform in mathematics below the international average in all TIMSS studies. In TIMSS, ratio and proportion problems are included under content domain of "Number". Percentages of both correct and incorrect responses as well as the percentages of responses of distractors of 15 ratio and proportion problems are investigated. The results reveal that, in all of the ratio and proportion problems the percentages of correct responses of eighth grade students from Singapore are much more higher than international average of percentages of correct responses. Moreover, Singaporean eighth grade students have the highest correct response percentages for some of the ratio and proportion problems.

When we compare the percentages of eighth graders from the United States and Turkey, it is seen that in all of the ratio and proportion problems except one, American eighth grade students display better performance than their Turkish peers. The correct percentages of eighth grade students of Turkey are either at or below than international average of correct percentages for all of the ratio and proportion problems except one. The in-depth analyses of the responses reveal that, there are many American and Turkish students those have difficulty in detecting multiplicative relationships. Similarly, a considerable amount of Turkish students think additively in ratio and proportion problems. Moreover, Turkish students operate ratio as a fraction, and it seems that they adopt only the part-whole meaning of the fraction. Although eighth grade students from Singapore are very successful in ratio and proportion problems, their performances in speed problems are outstanding.

The results of the current study are discussed based on the differences in mathematics curriculum and mathematics textbooks of these three countries. It is seen that performance differences of eighth graders of different countries may be attributed to differences in the mathematics curriculum and mathematics textbooks to a large extent. For instance, the great emphasize on the concept of "speed" and frequent use of models in Singapore mathematics textbooks, inexplicit definition of ratio in Turkish mathematics textbooks and insufficient explanations of the used strategies in ratio and proportion problems in Turkish mathematics textbooks are some of the discussion topics in order to explain the performance differences among eighth grade students of Singapore, the United States and Turkey.

Conference Fee

We ask the participants to support the conference proceedings with 25 Euros. It shall be paid on the conference by cash.

WELCOME TO BUDAPEST!