ProMath 2014 Abstracts

András Ambrus: Opening the problems is one step forward to reach more students

The Hungarian mathematics PISA 2013 results are showing a decreasing tendency. The number of best students – qualification 6 – low, and the number of students with very low achievement – 1 or below 1 - is increasing. The reasons may be complex, and I try to emphasize only one characteristics of Hungarian mathematics education, namely the high abstract level from early grades. In Hungarian mathematics textbooks, task-collections and between mathematics competition problems you may find problems only in closed form. Direct consequence is that most of the students can't start the solution of the task without the teacher's help, teacher's prompts. In our talk we will report about our experiences with middle school students aged 10-13 in a small town in South-Hungary. The results are clear: most of the investigated students could not do anything with the closed problem, but at the open version everybody could start, could make right solution steps forward. After analysing some problems from the point of view of the long term memory, we present different students' trials, ideas and make some suggestions how some steps forward to reach more students in Hungary in mathematics problem-solving teaching can we make.

Lars Burman: Teaching problem solving with problem sequences

There is a need to improve mathematical thinking and the quality of reflection and thought. This can be done by offering the pupils good questions and challenges, as well as possibilities to work with a somewhat more extended problem. It is preferable to divide the work with problems into steps and to give the pupils possibilities to discuss within the whole class and with the teacher between these steps. Thus, the pupils are able to receive exercise in handling with different kinds of problems. After that, they can be tested if they are able to solve certain problems individually and in groups.
I am working with pilot-tests using teacher-guided problem sequences in a lower-secondary school in Finland. When problem sequences are used, the pupils work partly in groups and partly individually, they discuss the results with the teacher, they receive new information and guide-lines and then they proceed. Up to now, the focus has been on the development of problem sequences and on the design of problems suitable for heterogeneous classes and for work in groups.

Montserrat Castelló-Badia and Loles González-García: Learning Mathematics: study of the impact of contextual factors according to the performance of students.

The international study TIMSS (Trends in Mathematics and Science Study) evaluates the mathematics and science performance of students in fourth grade of Primary Education and second of Secondary Education. Spain participates only in Primary Education.

Results from the TIMSS 2011 Report show that Spanish students are below the average of the countries evaluated in attitudes and strategies for learning mathematics. These results reaffirm the need to review what and how to teach in the field of mathematics in the Spanish educational system. The aim of present study is to identify the influence of personal factors on student performance in mathematics. Participants were 52 students in the last primary education year from a private publicly funded schools in Barcelona. They were divided into three groups according to knowledge of mathematics (high, medium and low).

Information was obtained through a background questionnaire designed based on the dimensions established by the TIMSS study, in which questions were asked about the sense of belonging at school, motivation, perception, anxiety, self-esteem and perseverance to study of mathematics. A four-point Likert scale was used: Strongly Disagree, Disagree, Agree and Strongly Agree. The study is a descriptive-interpretive case analysis design with qualitative and quantitative combined information.

We analyzed data using Spearman correlations, factor analysis and variance to determine the impact of gender and other variables on performance (high, medium, low). Preliminary results show significant differences between groups for motivation ($P < .028$), anxiety ($P < .038$) and self-esteem ($P < .002$). Significant sex differences in performance ($P < .04$), anxiety ($P < .01$) and self-esteem ($P < .001$) were also found.

The results will be discussed in terms of some educational implications for teaching and learning of complex content, such as problem solving.

Olive Chapman: Teachers' thinking and practices that support students' engagement in mathematical problem solving

Mathematical Problem-Solving Knowledge for Teaching [MPSKT] is more than knowledge of how to solve problems or competence in solving problems. This paper contributes to our understanding of this knowledge, which is important to help teachers to effectively support students' engagement in problem solving. The focus is on what constitutes MPSKT from a theoretical perspective and a practice-based perspective. The relationship between problem-solving proficiency and mathematical proficiency and a perspective and model of MPSKT teachers ought to hold to help their students to develop proficiency in problem solving are discussed based on a review of the literature on mathematical problem solving. Practice-based perspectives of MPSKT are discussed based on a study of secondary school mathematics teachers' thinking and teaching of problem solving. In particular, the study focused on contextual problems involving real or imagined situations and closed or open solutions used by the teachers and investigated the knowledge the teachers held and used, how they used it, and the role it played in shaping their teaching and students' engagement with contextual problems. The study produced four examples of practice-based MPSKT that supported different levels of problem-solving proficiency and different levels of student engagement in problem solving with contextual problems. Each example will be discussed in terms of its components, the importance of the interdependence among the components, and the nature of student engagement it afforded. The findings have implications for teacher education, for example, they provide a model to build on in designing and researching professional development programs to foster teaching for problem-solving proficiency. The examples can also form a basis to help teachers to understand the nature of this knowledge and how they can support and limit students' engagement and development of proficiency in problem solving.

Elçin Emre & Ziya Argün: Instructional design-based research on Problem Solving Strategies

Lately among mathematics educators, a tendency towards using problem solving as a significant component of the curriculum has been aroused. For learners, the need to become successful problem solvers has become a dominant theme in many international standards. Success of problem solving depends on two main factors related to solver's ability, one is choosing an appropriate strategy and the other is applying it properly. The main goal of this study is to find out the effect of the instructional design method on the enhancement of these abilities of students. Teaching sessions were applied to ten students who are in 11th grade, to teach them problem solving strategies which are working backwards, finding pattern, adopting a different point of view, solving a simpler analogous problem, extreme cases, make drawing, intelligent guessing and testing, accounting all possibilities, organizing data, logical reasoning. Our study is based on one-on-one (teacher-experimenter and student) design experiments where we conduct teaching sessions with a small group of students to study in depth and detail. We designed sessions for ten-week to teach 10 high school students 10 problem-solving strategies. Every week students worked on different type of problems but learning one problem solving strategy. However, we did not restrict students to solve the problems only with one strategy. In contrary, during the experiments, we encouraged students to solve problems using different strategies. Before and after the application of instructional design, 12 different problems were given to students. After the instructional design, we made interviews with students. We analyzed the solutions of the students by comparing each student's solution before and after the teaching sessions in terms of the ability to choose and apply the appropriate problem solving strategy. At the end of the analysis, we observed that the instructional design helped students on choosing and applying an appropriate strategy. Students became aware of the existence of several different problem-solving strategies and they discovered that they could use more than one problem solving strategies for one problem.

Eva Fülop: Teaching problem solving strategies in mathematics

This study uses the methodology of Design Based Research in search of ways to teach problem solving strategies in mathematics in an upper secondary school. Educational activities are designed and tested in a class for four week. The design of the activities is governed by three design principles which are based on Variation Theory. Using pre- and post-tests, we compare the development of the students´ conceptual and procedural abilities with a control group. We use the post-test to investigate the students´ use of problem solving strategies. This study aims to contribute to understanding how teaching of problem solving strategies and strategy thinking in mathematic can be organized in a regular classroom setting and how this affect students´ learning in mathematic.
Key Words: strategy thinking, variation theory, design based research, problem solving strategy, classroom teaching

Introduction

For the past 40 years, problem solving and strategic thinking has emerged as one of major concerns at all levels of school mathematics. And at the same time in a world that is becoming more turbulent and characterized by rapid technological innovations, shifting political alliances, and emerging economies, strategic thinking and problem solving plays a crucial role in both our everyday- and professional life. (NCSM, 1977; Mason 1986; Sloan, 2006; Goldman, 2012) But how do we successfully learn how to solve problems? To be a good problem solver is it important to be able to think strategically? If so, what is most essential when we are learning to think strategically and what learning approaches could be used to become a successful at it? How do managers get their organizations continually to adapt and change in new directions and to generate the momentum needed to propel innovation? Mathematics teachers can ask a similar question: How can we as teachers get our students to continually adapt and move in new directions when this is needed in mathematical problem solving? If a task is difficult or we get stuck knowledge about problem solving strategies can help us find new approaches for solving this problem. In other words strategies can help us know what to do when we don´t know what to do.

The study presented in this paper contributed to this body of research. The aims of the study is to examine ways of teaching problem solving strategies in mathematics in upper secondary school through specially designed activities and how this affects students' problem solving, conceptual and procedural ability. This study was conducted as a Design Based Research (DBR) and used teaching intervention intended to support 16 and17 years old students to identify problem solving strategies and to experience strategy thinking. To frame a systemic research agenda that addresses issues of sustainability and usability of teaching problem solving strategies the teaching intervention was designed to fit into the regular teaching and without altering the mathematical content but adding learning of strategy thinking. This paper begins by clarifying the nature of strategy thinking, of the concept strategy in professional life and in school mathematics through presenting a historical overview of perspectives of the concept strategy and of differences of concepts strategy, method and algorithm. Next we describe the methods and results from our study. And finally we put forth some recommendations for supporting the development of strategy thinking on the individual and the classroom levels.

Günter Graumann: Calendars in different cultures and its importance for school

Following my presentation of the last ProMath conference I like to present this year another problem field connected with astronomy namely calendars. With a calendar we all are confronted permanent and think of our calendar to be self-evident. But in our global connected world we also hear from other calendars or rhythms, especially in the context of religion. So it is good for children at school to get to know about the background of calendars as well as to fight with problems within the field of calendars.

First of all we have to pick up (in books or in the internet) some knowledge about astronomical facts and the origin of different calendars whereat we can see how the problem to bring times which are not whole numbers into special rhythms of whole numbers was managed in history and how we can present any rational number in a rhythm of whole numbers. Secondly we can compute with the Islamic calendar which is a pure moon-calendar and find out why and in which way the Islamic festivities move in our solar calendar. Furthermore we will discuss the Christian festivities and its differences in catholic/protestant churches and the Russian-orthodox church. Especially we have to work on the problem of determining the date of Easter (the "computus ecclesius" as it was named in the middle ages).

If the time is not running away then finally we can have a look at different phases of the moon and the angular velocity of the moon and the stars in the night as well as the division of the whole world into time zones.

Riina Harri, Markus Hähkiöniemi and Jouni Viiri: The relationship between the use of representations and peer interaction in mathematics classroom

The aim of this study is to investigate how the use of mathematical representations and peer interaction are related. Pupils´ use of representations and peer interaction has previously been in the focus of the mathematics education research. Researchers have explored ways to coordinate individual and social aspects in analyzing mathematics learning (e. g. Francisco 2013, Tabach, Herskowitz, Rasmussen & Dreyfus 2014). However, the relation between the use of representations and group interaction has barely been investigated.

Theoretical framework of this study is sociocultural. Sociocultural theory of learning asserts that one can understand individual learning by studying how the social environment is organized and how individuals participate in social practices (Vygotsky 1978). This case study will be situated at a Finnish elementary school in South Finland during the school years 2014–2015 and 2015–2016. In year 2014–2015 the 16 participants are in third grade (age 9).

The data will be gathered by videotaping of four small groups, four pupils in each one, in the problem solving settings during two week long periods at a time. The study will consist of four periods per a term, two of them during the autumn (2015 and 2016) and two during the spring (2016 and 2017). In addition to video and audio recordings, all written work of pupils will be copied for as a data source.

Analysis will rely primarily on full transcripts and secondarily on the written work of pupils and video recordings, which allow the researcher to consider mainly the participants talk, but also their gestures, use of artefacts, drawings and physical objects. In this study, representations are categorized into three systems of representations. According to Bruner (1966) the categories are: enactive, iconic and symbolic. The use of representations will be analyzed by applying RBC + C –model by Tabach et al. (2014). The interaction of small groups will be analyzed according to three kinds of talk identified by Mercer (1996). The study will take into account the context of the whole class as well. The ways in which ideas evolve and spread across the individual, small group and a whole class will be analyzed using the notion of knowledge shifts by Tabach et al. (2014).

References:

Bruner, J. 1996. *Towards a theory of instruction*. Cambridge, MA: Harvard University Press.

Francisco, J., M. 2013. Learning in collaborative settings: students building on each other´s ideas to promote their mathematical understanding. *Educational Studies in Mathematics*, 82 (3), 417–438.

Mercer, N. 1996. The quality of talk in children´s collaborative activity in the classroom. *Learning and Instruction* 6 (4), 359–377.

Tabach, M., Herskowitz, R., Rasmussen, C. & Dreyfus, T. 2014. Knowledge shifts and knowledge agents in the classroom. *The Journal of Mathematical Behavior*, 33, 192–208.

Vygotsky, L. S. 1978. *Mind & society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Henna Heikkinen: Promoting Exploratory Teaching in Mathematics: A Design Experiment on a CPD course for Teachers

In this talk we present a design experiment on a continued professional development (CPD) course for mathematics teachers. This research is joint work with Peter Hästö, Vuokko Kangas, and Marko Leinonen, of the University of Oulu.

The general themes of the course, *Introduction to Exploratory Learning in Mathematics*, are teaching methods that promote active learning and exploratory learning environments. The course consists of one-day, on-site training and is aimed at both elementary and high school mathematics teachers. It is the first part of a larger CPD unit; however, the other parts of the unit are not part of the design experiment, and are not considered further here.

The three main goals of the course are:

1) To impact positively on the participants' attitudes towards and beliefs about problem solving and its role in mathematics teaching.
2) To provide examples of concrete ways to implement problem solving and co-operative learning in their mathematics teaching.
3) To promote the participants' self-confidence in using exploratory teaching.

The goal of the design experiment is to study the factors furthering and impeding the realization of these course goals, and to develop the course to be maximally effective in achieving them.
The design experiment paradigm consists of implementing multiple iteration of a teaching cycle. In each cycle, the course is delivered, data is collected and analyzed, and changes are made to the course content accordingly. In a design experiment, one also aims to develop theory at the same time. In this presentation we describe this iterative process of developing the course.
The course has been organized four times with about a one-month analysis period in-between. The design experiment thus consists of three iterations. The first implementation was based on an earlier pilot course. In this talk we describe the data analysis and course revision process in these iterations. We present the changes to the course design, and our preliminary findings about factors affecting the course's effectiveness in achieving its goals.

Markus Hähkiöniemi: Students' responses to teachers' explanation requests during problem solving

In mathematical problem solving, it is important that in addition to building solutions to problems, students engage in explaining their solutions. The teacher can press students to explain by questioning. Yet, students' can explain different things. Procedural explanations describe how something is done mathematically whereas conceptual explanations give mathematical reasons. For example, a procedural explanation describes the steps in a solution to a problem and conceptual explanation offers reasons why the solution is valid.
The aim of this study is to find how frequently students give conceptual and procedural explanations when a teacher asks them to explain. Data was collected by video recording 29 Finnish pre-service mathematics teachers' lessons in secondary and upper secondary schools. The lessons included a phase during which students solved problems and the teacher circulated guiding them. The lesson videos were coded for teachers' requests for explanation and for types of students' explanation (conceptual, procedural or no explanation).
According to the results, often students did not give mathematical explanations even though a teacher asked for it. When students actually explained something, the explanation was more often procedural than conceptual. The frequency of student explanations was related to frequency of teacher questions. Analysis of the explanation situations offers more insights into the conditions in which students offered different types of explanations.

Peter Hästö: Development of mathematical problem solving at the university level

In this talk I present work with Henry Leppäaho (Jyväskylä) and Raimo Kaasila (Oulu) on the development during a one-semester MPS-course of prospective secondary school teachers' views on mathematical problem solving. The study is based on a collection of reflective essays (n=10) written by the students in the course. Using a qualitative, data-driven analysis we identify changes in students' conceptions and stated actions, as well as students' views on the reasons for these changes.

Thomas Jahnke: Mathematics of Participation and the Theorem of Arrow
Examples and ideas to deal with the theme Social Choice and Welfare as an understandable and important pearl of discrete mathematics in school.
One of the mainpoints is to formulate visualize and prove the Theorem of Arrow in different ways.

Meira Koponen: Teacher as the instructor of the reflection phase in the problem solving process

Problem solving process can be divided into different phases, where the reflection phase is the last phase of solving the problem, taking place after the solution has been obtained. During the reflection phase there is an opportunity to reflect, review and analyze one's solution and make generalizations.

In this presentation I present some of the finding of my master's thesis, where I analyzed three primary school teachers' instruction during the reflection phase. The teachers were a part of a larger study and they had taught several problem solving classes during the study. The instruction in the reflection phase was divided into three categories: the 'mathematics' of the problem and the mathematical context, problem solving strategies and the beliefs and attitudes of the students.

The instruction in the reflection phase should mirror the objectives of the lesson. It seemed that the teachers who had a specific objective in mind during their instruction, gathered students' ideas and formulated the results in a systematic manner in the final classroom discussion. However if a clear objective wasn't detected, the instruction during the reflection phase was only limited.

Ana Kuzle: Problem Solving as an Instructional Method: The Case of Strategy-Open Problem "The Treasure Island Problem"

Problem solving is not only an instructional goal, but also an instructional method in which teachers use problem solving as a primary means to teach mathematical concepts and help students synthesize their mathematical knowledge. As an instructional method it can be used to build new mathematical knowledge, to solve problems that arise in mathematics and in other contexts, to apply and adapt a variety of problem-solving strategies, and to monitor and reflect on the mathematical problem-solving processes. On one exemplary problem this instructional method will be typified under technological considerations. Here technology allowed preservice teachers to understand, approach and solve a problem than it would be possible without it. Moreover, it allowed them to experience genuine problem solving and the joy of discovery in Pólya's sense. Implications for mathematics instruction at the secondary and tertiary level will be given at the end of the report.

Henry Leppäaho: Mathematics student teachers' experiences from an upper secondary school level problem-solving task

In this research mathematics student teachers had to solve a problem-solving task in the Finnish matriculation examination. After the solving process the students had to assess, for example: What did they think about the task? What was the level of the difficulty in the task? How was their own ability to solve the problem and to teach it? There is a general belief that the university mathematics develops students' mathematical thinking in such a way that the matriculation examination-level tasks are too easy "school math" for them, which is not reasonable to be studied at the university-level. The findings of this study reveal that this belief was not always supported.

John Mason: On Being Stuck On A Mathematical Problem:
what does it mean to have something come-to-mind?

Everyone gets stuck sometimes, and it can be frustrating, even though being stuck is an honourable state because that is when it is possible to learn about oneself. People are usually eager to get unstuck, to locate and enact some hopefully helpful action, without attending to how they got stuck in the first place, nor how they got going again. I propose to dwell in the states of becoming and being stuck, and to use this as a springboard to examine and amplify the notion of 'having some possibility come to mind' as a means to get unstuck. This will include an expansion of the notions of *system 1* and *system 2* (automatic-habitual reaction and considered response) to take account of affect and of 'educated awareness'. My method will be as phenomenological as possible, drawing on specific examples from my own experience, but hoping to resonate with the experience of participants.

Erkki Pehkonen: Open problem solving in elementary mathematics teaching:The Finland – Chile research project

Here we will describe a research project, implemented in 2010–13 in the Department of Teacher Education at the University of Helsinki where we try to parallel the Finnish and Chilean teaching practices in mathematics when using open problems once a month beside the ordinary teaching. A brief description on the implementation of the project will be given. The most important background studies are mentioned – students' conceptions on mathematics, students' drawings on a mathematics lesson, students' mathematics knowledge and problem solving skills, teachers' conceptions on problem solving. Additionally it is given a list of existing publications so far in English.

Päivi Portaankorva-Koivisto, Erkki Pehkonen & Anu Laine: Different instructions and different solutions in the case of the pentomino problem

In this presentation, it is discussed some results of a larger research project called "*On the development of pupils' and teachers' mathematical understanding and performance when dealing with open-ended problems in two countries, Finland and Chile*" (2010–13), which was financed by the Academy of Finland (project number 1135556). In this study we concentrated on one open-ended problem, "Pentomino-problem", which was conducted at the 5th grade (age 10-11) of the Finnish comprehensive school.

A pentomino is a geometric figure that is formed by joining five congruent squares together along their edges. Two pentominoes are different if they are not congruent; this means, that one pentomino cannot be made to match with the other by flipping or by rotating. (Cowan, 1977) Manipulating pentominoes can help pupils to improve their geometric learning, such as spatial reasoning, problem solving and visual skills (Yang & Chen, 2010). With the help of pentominoes we can examine the "concepts of congruence, similarity, transformations (flips, turns, and slides), tessellations (tiling), networks, perimeter, area, and volume in a problem-solving, cooperative learning environment" (Onslow, 1990, 5).

The problem was set as follows, but each participating teacher could organize the teaching arrangements independently:

1) Build as many pentominoes as you can by using multilink cubes. Draw the pentominoes you found on a squared paper. In total, there are 12 different types of pentominoes. Can you find them all?
2) Try to put together three or more pentominoes. How many different rectangles can you construct? Draw these rectangles on your paper.

The data used in this study comprises the video recordings from the lesson and the pupils' working papers collected from six participating classes in Finland. The research questions were: (a) What kind of teaching arrangements did the teachers design? (b) What kind of results did the pupils perform?

The study yielded that some teachers instructed their pupils to concentrate on the process: "Number the pentominoes you have drawn, so that we know the order in which they were found." Some teachers relied on the systematics: "Draw on your paper an arrow to indicate what cube was moved next." Some teachers were more open with the documentation: "You can outline the shape of your pentomino by following the edges of the piece." Pupils' results were different in every class. Those using the systematics found all the 12 pentominoes and did not make errors with transformations. In one class, the teacher had constructed all the pentominoes by herself and pupils could correct their shapes after comparing them with teacher's model pentominoes. If teachers have given minimal instructions to pupils, some pentominoes were left to tackle, some congruent shapes were regarded as different, and some pupils were not able to sketch their pentominoes on paper.

Cowan, R. A. (1977). Pentominoes for fun and learning. *Arithmetic Teacher*, 24(3), 188–190.
Onslow, B. (1990). Pentominoes revisited. *Arithmetic teacher*, 37(9), 5–7.
Yang, J. C., & Chen, S. Y. (2010). Effects of gender differences and spatial abilities within a digital pentominoes game. *Computers & Education*, 55(3), 1220-1233.

Benjamin Rott: Rethinking Heuristics – Characterizations and Vignettes

The concept of "heuristics" or "heuristic strategies" is central to (mathematical) problem solving and related research; however, there is no generally accepted definition of this term. Trying to clarify the concept might help avoiding misunderstands and difficulties in dealing with studies that use different terms meaning the same concepts or that use the same terms meaning different concepts.
Building on last year's work, I'm going to investigate the consequences of using different characterizations of "heuristics" on vignettes (= short, completed scenes) of problem solving attempts. The conceptualization of "heuristics" has a significant influence on the types and numbers of perceived heuristics, which in turn affects empirical studies that identify and analyze heuristic strategies.
The goals of this research are a clarification of the term and suggestions for the use of it in future research.
Key Words. Heuristics, Problem Solving, Theoretical Foundation

Pavel Shmakov: INTELLECTUAL PASSION on example of CheCha maths: affect and problem solving in math education

Intellectual passion (IP) - emotional (motivational) state while fully absorbing pastime in a particular subject area. Passion is like to affect, and can be expressed only in relation to a single selected object, action. Not surprisingly, one can find the following synonyms for passion: excitement, vehemence, takeoff. To distinguish the concepts of IP and interests , consider some of the definitions of interest: «interest - consumerism attitude or motivational state , encourage a cognitive activity unfolding mainly domestically». «Interest - the emotional state associated with the implementation of cognitive activity and this activity is characterized by incentive». Work for a rapt pupil turns into a need, it is vital daily celebration parallel awakening enthusiasm, encouragement, inspiration, energy, activity. The world around the pupil (social environment, school environment, family) is building the basis of their interests, which may be diverse. However, carried away by any one selected direction (business, a certain domain of knowledge), the pupil goes from calm observation to the state of elation and euphoria. Passion allows to experience pleasure from the process of labor and to be excited about the results, and to make an important discovery for yourself and share it with the world. Passion for anything is peculiar to children age. IP can be developed in the course of an intellectual activity.

Different methods and conditions can be used for the development of IP:

1. Positive emotional and professional support of pupil. Emotionally supportive environment can be created by parents, friends, teachers. Professional support implies professionalism of a supervisor in the intellectual domain, which is selected by pupil himself. Teachers themselves act in most cases as chiesf scientists at pupils scientific conferences. It is also important to engage scientists who can recruit students to ”real” research ”today or tomorrow”.
2. The presence of a variety of areas at the school to help finding ones own specialization and to see progress in other areas. The presence of an enthusiastic little "mathematician" and enthusiastic little "historian" in the same class makes it possible to create a unifying atmosphere of cognitive activity in the field of history for "mathematician" and in mathematics for "historian". A Fascinated and accessible "mathematician" may be able to explain the complexity and inspiration of mathematics to a "historian", and to do it even better than an adult teacher.
3. Usage of publicly available methods, that do not depend on intellectual abilities, and at the same time reveal these opportunities , intellectual techniques developing such passion, such as the inclusion of humor in the subject area or use Cheerful and Challenging mathematical method ("wrapping" mathematical "nuts" in a fabulous wrap , and so that the " nut" has both an easy and a difficult solution). This is especially important for those students who are not yet interested in the subject, who should be captured by it and needing a joyful atmosphere and creative mood.

CheCha mathematics is a teaching approach that is built around math problems that are Cheerful and Challenging for the pupil at the same time.

Isabel Velez & João Pedro da Ponte: How third grade teachers promote the understanding of their students representations

The role that representations assume in teachers’ practices have a strong influence in students’ learning (Stylianou, 2010). To help students to understand symbolic representation, several authors (Acevedo Nistal, Doreen, Clarebout & Verchaffel, 2009; Webb, Boswinkel & Dekker, 2008) recommend that teachers should encourage their students to begin by creating their own representations.

In this presentation we analyze the practices of two third grade teachers, Sara and Sofia, who work in two schools that belong to the same cluster in the surroundings of Lisbon, in Portugal. Our goal is to understand how these teachers explore the tasks that they suggest to their students, particularly regarding how they promote the learning of representations by their students and the development of their reasoning. Data were collected through video recording of lessons, including the moments of interaction between teacher and student and student paper records. Data were analyzed taking into account the class moments referred by Ponte (2005): introduction, students’ autonomous work, and whole class discussion of results.

The teachers proposed the following problem: “In a theater play performed by third grade students, João, Pedro and Ulisses wanted to be the King. On the other hand Ana, Inês and Estrela wanted to play the role of Queen. How many pairs of King/Queen could be formed?”

During the introduction of the task both teachers paid much attention to students’ understanding of the problem. While Sara asked some students to explain in their own words what they have understood from the problem statement, Sofia questioned her students about it. The two teachers had to deal with the students’ difficulty related to the interpretation of the word “pair”. They solved this quickly with the help of their students who suggested another meaning for the word (“couple” in Sara’s class and “a group of two” in Sofia’s).

At the end of the introduction, the students tried to find an answer as quickly as they could and they tried to tell it to their teachers. Both teachers reinforced that they needed to write it down and to justify their answers so they could discuss it with their colleagues (Sara says: “So have you write it?! OK… I do not want you just to tell me! I want you to explain it to me and tell me which the pairs are! How many are they?! Why?!”; Sofia says “I do not want to see [written] only the answer (... !) I will accept your answer ... When you justify why and which are the couples!”
During the students’ autonomous work both teachers focused in the different types of representation used by students and questioned them about the representations that they were using (“Why did you do that? How did you saw it?”). They also praised the students’ work (“Very good!”, “I really liked the representation that you chose!”) and as students eared the compliments made to their colleagues, they tried to make an effort to get the right answer with an interesting representation. In Sara’s class the students mainly used different types of diagrams and in Sofia’s some students tried to solve the problem by using pictorial representations and started to feel discouraged with the slowness of process. Noticing that, Sofia felt the need to support students to search for a more suitable representation and decided to conduct a brief discussion with the class about the representations used by the students so far. Regarding students’ reasoning, in Sara’s class although the students used mainly diagrams some of them used letters and numbers in organized schemes, while others, although using similar schemes, showed random strategies.
During the whole class discussion of results, Sara asked three students to show and explain their representations and strategies to their colleagues. That way, Jonas showed that he wrote down every pair Queen/King and Sara asked him “Why did you made a scheme like that one?”, Mauro drew a line between a boy and each girl, and explained it (with Sara’s’ help), showing that each boy can make a pair with three different girls, and Mara explained that she used the number “3” to symbolize each set of 3 and she added everything (3+3+3). Asking as a starting point for an incomplete solution from a student (that had write down the pairs), Sofia discussed collectively the task with her students and she gave some examples based on previous students’ representations (she had established a connection between boys and girls). At the end of the whole class discussion both teachers questioned their students so they could discover and understand a formal representation that could be used in this situation.
The way teachers organized students’ autonomous work and whole group discussion of results, and how they acted proved to be crucial in the emergence of different types of representation as teachers led students to search and use different representations, questioned them about the used representations and encouraged them to establish a connection between the different representations. While encouraging the use of different types of representations, teachers preferred that the students to use less the pictorial representations, privileging schemes and symbolic representations. However, when they asked students to write all the possible pairs they may have influenced most of them to use schemes, avoiding symbolic representations.

References

Acevedo Nistal, A., Dooren, W. V., Clarebout, G., Elen, J., & Verschaffel, L. (2009). Conceptualising, investigating and stimulating representational flexibility in mathematical problem solving and learning: a critical review. ZDM, 41(5), 627–636.

Ponte, J. P. (2005). Gestão curricular em Matemática. In GTI (Ed.). O professor e o desenvolvimento curricular (pp. 11-34). Lisboa: APM.

Stylianou, D. A. (2010). Teachers’ conceptions of representation in middle school mathematics. Journal of Mathematics Teacher Education, 13, 325-343.

Hanna Viitala: Emma and Nora solving a PISA problem
I will introduce you two Finnish girls solving one PISA problem (Holiday). I will combine data from two videos that I have from both pupil. In the first video pupil solves the PISA problem in classroom situation, and in the second video pupil explains her thinking using the first video as stimuli in a stimulated recall interview. Finally I will examine what kind of similarities and/or differences are there in the problem solving of these two girls. Data about affect and background is also collected through interviews. The girls are both from different schools and regions in Finland.

Betul Yayan: Investigating Turkish students' problem solving skills in PISA from the view of mathematics curriculum reform

The Programme for International Student Assessment (PISA) conducted by Organisation for Economic Co-operation and Development (OECD) in member and non-member nations is a triennial international survey which aims to evaluate skills and knowledge of 15-year-old students in mathematics, science, and reading. In mathematics area, the focus of PISA assessment is on mathematical literacy which is defined as individual's capacity to formulate, employ and interpret mathematics in real world problems presented in a variety of contexts. The most fundamental constructs of the mathematical literacy are the mathematical thought and action that students use to solve the problem and the processes that the problem solver uses to construct a solution for the problem. Since PISA is an ongoing assesment, it helps to monitor trends in students' acquisiton of knowledge and skills within each country. In this sense, the mathematics performance of Turkey is noted as steadily improving from PISA 2003 to PISA 2012. One of the development and improvement efforts presumably playing an active role in this observed improvement was the currciculum reform that has been implemented since 2006 in Turkey. The aim of the present study was to compare mathematics literacy performances of 15 year-old Turkish students participated in PISA 2003 and PISA 2012. Furthermore, the percentages of Turkish students' correct responses for the released items were compared with the OECD average and these comparisons were interpreted within the context of mathematical processes and capabilities required to solve the released items. The results revealed that there is decrease in the percentages of Turkish students attaining the lower proficiency levels indicating that fewer Turkish students were able to solve only very direct and straigforward mathematical problems in PISA 2012. On the other hand, there were minor increases in the percentages of Turkish students attaining Level 2, 3, and 4. In other words, there were more Turkish students who could employ basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers, who could interpret and use representations based on different information sources and reason directly from them to solve the problems and finally who could select and integrate different representations including symbolic representations, linking them directly to aspects of real-world situations. Among the item pool of released items in PISA 2012 the simplest item for Turkish students was the item in which students have to read height of the bar to obtain the response on the other hand the most difficult item was related to calculation of percentage within a given real world situation. The observed differences between Turkish students' mathematical literacy in PISA 2003 and 2012 were interpreted in the light of focus and content of the new mathematics curriculum.

Gönül Yazgan-Sağ & Ziya Argün: Turkish gifted students views about mathematical problems solved in classroom: Are these problems challenging or not?

There are different descriptions of giftedness in mathematics area. Generally literature agrees that these students think qualitatively different from average students (Reed, 2004). They also tend to get bored in mixed classrooms. Gifted students should engage with challenging problems (Diezman & Watters, 2004). Therefore it is very important to choose appropriate problems in order not to lose these students in the classrooms.

The purpose of this research was to investigate Turkish gifted students views about problems which they come across in their mathematics classrooms. The data were gathered from semi-structured interviews. We interviewed with three 10th grade secondary gifted students. Each interview took about 40 minutes. We asked questions about their opinions about challenging problems and their mathematics lessons. We analyzed the data using by content analysis and constant comparative method (Patton, 2002; Strauss & Corbin, 1998).

As a consequence of this research we have the following results about problems which they encounter in their lessons. Firstly gifted students think that their mathematics lessons are boring. Also they find problems routine and easy. These students want to solve problems that include everyday life situations. Besides gifted students prefer to solve authentic and challenging problems make them thinking differently.

Bernd Zimmermann: 30 years Hamburg Model to foster gifted pupils and some possible influence on normal classroom teaching

The main philosophy of and some experience (from more than 30 years) with the Hamburg Model to foster mathematically gifted pupils will be presented. This experience will be complemented by impressions from normal classroom teaching and the development of a textbook, which were triggered by this project.